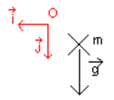
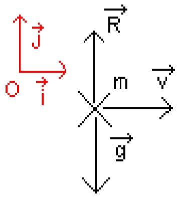
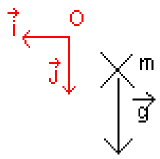
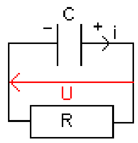
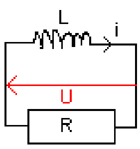
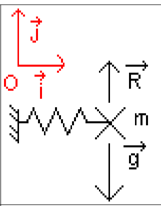
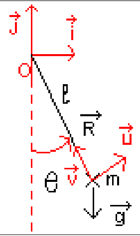
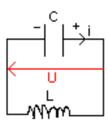
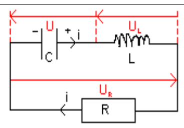
Tableau récapitulatif des équations différentielles appliquées à des cas physiques *(© Basnary S.)*

Observable	***Schéma***	***Formule et équation différentielle***	***Solution et relation***
1er ordre simple		$y' = f(x) \iff dy/dx = f(x)$	$y(x) = F(x) + k$ avec $F(x)$ primitive de $f(x)$
Chute libre sans frottement Vitesse $v(t)$		$\Sigma \vec{F}_{ext} = m\dfrac{d\vec{v}}{dt} \Leftrightarrow \begin{cases} dv_x/dt = 0 \\ dv_y/dt = g \end{cases}$	$\begin{cases} v_x(t) = v_{0,x} \\ v_y(t) = g\,t + v_{0,y} \end{cases}$
1er ordre + 2nd membre		$y' + a\,y = b \iff dy/dx - (-a)\,y(x) = b$	$y(x) = k\,e^{-ax} + b/a$ (a en s^{-1} si x en s)
Radioactivité Nbr de noyaux $N(t)$	Utiliser l'activite $A(t) = \lambda\,N(t)$	$A(t) = -\dfrac{dN(t)}{dt} \Rightarrow \dfrac{dN(t)}{dt} + \lambda\,N(t) = 0$	$N(t) = N_0\,e^{-\lambda t}$ et $\lambda \times T_{½} = \ln 2$ λ : constante (en s^{-1}) et $T_{½}$: période (en s)
Roulement avec Frottement vitesse $v(t)$		$\Sigma \vec{F}_{ext} = m\dfrac{d\vec{v}}{dt} \Rightarrow \begin{cases} \vec{P} + \vec{R} = \vec{0} \\ -\alpha\vec{v} = m\dfrac{d\vec{v}}{dt} \end{cases}$	$\dfrac{dv}{dt} + \dfrac{\alpha}{m}v = 0$ $v(t) = v_0\,e^{-\frac{\alpha}{m}t}$ $(\dfrac{\alpha}{m}$ en $s^{-1})$
Chute libre + frottement vitesse $v(t)$		$\Sigma \vec{F}_{ext} = m\dfrac{d\vec{v}}{dt} \Rightarrow \begin{cases} -\alpha\,v_x = m\dfrac{d\,v_x}{dt} \\ mg - \alpha\,v_y = m\dfrac{d\,v_y}{dt} \end{cases}$	$\begin{cases} v_x' + \dfrac{\alpha}{m}v_x = 0 \\ v_y' + \dfrac{\alpha}{m}v_y = g \end{cases} \Leftrightarrow \begin{cases} v_x(t) = v_{0,x}\,e^{-\frac{\alpha}{m}t} \\ v_y(t) = k\,e^{-\frac{\alpha}{m}t} + \dfrac{mg}{\alpha} \end{cases}$ $k = (v_{0,y} - \dfrac{mg}{\alpha})$
Circuit RC Tension $U(t)$ Courant $i(t)$		$\begin{cases} q(t) = C\,U(t) \\ i(t) = \dfrac{dq}{dt} \\ -R\,i(t) = U(t) \end{cases} \Rightarrow -R\dfrac{d\,C\,U(t)}{dt} = U$	$\dfrac{dU}{dt} + \dfrac{1}{RC}U = 0$ $U(t) = U_0\,e^{-\frac{t}{RC}}$ $(RC$ en $s)$
Circuit RL Tension $U(t)$ Courant $i(t)$		$\begin{cases} U(t) = L\dfrac{d\,i(t)}{dt} \\ -R\,i(t) = U(t) \end{cases} \Rightarrow U = -\dfrac{L}{R}\dfrac{dU}{dt}$	$\dfrac{dU}{dt} + \dfrac{R}{L}U = 0$ $U(t) = U_0\,e^{-\frac{R}{L}t}$ $(\dfrac{R}{L}$ en $s^{-1})$
2ième ordre		$y'' + \omega^2\,y = 0 \iff d^2y/dx^2 + \omega^2\,y = 0$	$y(x) = \lambda\cos(\omega x) + \mu\sin(\omega x)$ $y(x) = A\cos(\omega x + \varphi)$ (ω en s^{-1}, φ en rad)
Ressort horizontal position $x(t)$		$\Sigma \vec{F}_{ext} = m\dfrac{d^2\vec{x}}{dt^2} \Rightarrow \begin{cases} \vec{P} + \vec{R} = \vec{0} \\ -k\,\vec{x} = m\dfrac{d^2\vec{x}}{dt^2} \end{cases}$	$\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ $(\omega^2 = \dfrac{k}{m})$ $x(t) = x_{max}\cos(\omega t + \varphi_0)$ $(k$ en $kg\,s^{-2})$
Pendule $\theta(t)$ Utiliser Z_{OM} et $\Sigma \mathbf{F}_{ext} = m\mathbf{a}$ dans le repère de Frénet		$Z = l\,e^{i(\theta - \frac{\pi}{2})} \Rightarrow Z' = l\,\theta'\,e^{i(\theta - \frac{\pi}{2})} \Rightarrow v = l\,\theta'$ $\vec{P} + \vec{R} = m\vec{a}$ $\vec{a} = \dfrac{dv}{dt}\vec{u} + \dfrac{v^2}{l}\vec{v} \Rightarrow \begin{matrix} -mg\sin(\theta) = m\,dv/dt \\ R - mg\cos(\theta) = v^2/l \end{matrix}$	L'angle $\theta(t)$ est faible soit $\sin(\theta) \sim \theta$ $-mg\,\theta = m\dfrac{d}{dt}(l\,\theta') \Rightarrow -g\,\theta = l\dfrac{d^2\theta}{dt^2}$ $\Rightarrow \dfrac{d^2\theta}{dt^2} + \dfrac{g}{l}\theta = 0$ $(\omega^2 = \dfrac{g}{l}$ en $s^{-2})$
Circuit CL Tension $U(t)$ Courant $i(t)$		$\begin{cases} q(t) = CU \\ i(t) = dq/dt \\ -U = L\,di/dt \end{cases} \Rightarrow -U = L\dfrac{d}{dt}(\dfrac{d}{dt}C\,U)$	$\dfrac{d^2U}{dt^2} + \dfrac{1}{LC}U = 0$ $(\omega^2 = \dfrac{1}{LC})$ $U(t) = U_{max}\cos(\omega t + \varphi)$
2ième ordre: cas général		$y'' + a\,y' + \omega^2\,y = 0$ (a et ω en s^{-1})	Dépend des racines de $r^2 + ar + \omega^2 = 0$
$x(t)$: Ressort Horizontal + frottement		$\Sigma \vec{F}_{ext} = m\dfrac{d^2\vec{x}}{dt^2} \Rightarrow -\alpha\dfrac{dx}{dt} - k\,x = m\dfrac{d^2x}{dt^2}$	$\dfrac{d^2x}{dt^2} + \dfrac{\alpha}{m}\dfrac{dx}{dt} + \dfrac{k}{m}x = 0$
Circuit RLC Tension $U(t)$ Courant $i(t)$		$U_R(t) + U(t) + U_L(t) = 0$ $U_R = Ri,\ i = d(CU)/dt,\ U_L = L\,di/dt$	$RC\,dU/dt + U + L\,d/dt\,C\,dU/dt = 0$ $\Rightarrow LC\,d^2U/dt^2 + RC\,dU/dt + U = 0$